

**RAIGANJ UNIVERSITY  
DEPARTMENT OF MATHEMATICS**

**SYLLABUS FOR MATHEMATICS  
M. Sc.**

**CBCS FORMAT  
w.e.f. the academic session 2017 - 2018.**

**RAIGANJ UNIVERSITY  
Raiganj, Uttar Dinajpur  
West Bengal, India.**

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# **SEMESTER 1**



# Chapter 1

## SEMESTER 1

### 1.1 C. P.- 1. Linear Algebra:

Marks 20(Internal Assessment (I.A.))+55(Semester Examination (S.E.)) [Credit 6]

#### 1.1.1 Review of Vector Spaces

Vector spaces over a field. Subspaces. Sum and direct sum of subspaces. Linear span. Linear dependence and independence. Basis. Finite dimensional spaces. Existence theorem for bases in the finite dimensional case. Invariance of the number of vectors in a basis. Dimension. Existence of complementary subspace of any subspace of a finite dimensional vector space. Dimensions of sums of subspaces. Quotient space and its dimension.

#### 1.1.2 Matrices and Linear Transformations

Matrices and linear transformations, change of basis and similarity. Algebra of linear transformations. The rank-nullity theorem. Change of basis. Isomorphism Theorems. Dual space. Bi-dual space and natural isomorphism. Adjoint of linear transformations. Eigen values and eigenvectors of linear transformations. Characteristic and minimal polynomials of linear transformations, Cayley-Hamilton Theorem. Annihilators. Diagonalization of operators. Invariant subspaces and decomposition of operators. Canonical forms.

#### 1.1.3 Inner Product Spaces

Inner product spaces. Cauchy-Schwartz inequality. Orthogonal vectors and orthogonal complements. Orthonormal sets and bases. Bessel's inequality. Gram-Schmidt orthogonalization method. Hermitian, Self-Adjoint, Unitary, and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms, real quadratic forms.

### 1.1.4 Books for Reference in Linear Algebra

1. Friedberg, S. H., Insel, A. J. and Spence, L. J., Linear Algebra, Prentice Hall of India, Fourth Edition.
2. Kumaresan, S., Linear Algebra, A Geometric Approach, Prentice Hall of India, Fourth Printing.
3. Artin, M., Algebra, Prentice Hall of India.
4. Halmos, P. R., Finite Dimensional Vector Spaces, Springer.
5. Roman, S., Advanced Linear Algebra, Springer.
6. Curtis, C. W., Linear Algebra : An Introductory Approach, Springer (SIE).
7. Hoffman, K. and Kunze, R., Linear Algebra, Prentice Hall of India.

## 1.2 C. P.- 2. Abstract Algebra: Marks 20(I.A.)+55(S.E.) [Credit 6]

### 1.2.1 Groups

Review of basic concepts of Group Theory: Lagrange's Theorem, Cyclic Groups, Permutation Groups and Groups of Symmetry:  $S_n$ ,  $A_n$ ,  $D_n$ , Conjugacy Classes, Index of a Subgroup, Divisible Abelian Groups. Homomorphism of Groups, Normal Subgroups, Quotient Groups, Isomorphism Theorems, Cayley's Theorem. Generalized Cayley's Theorem, Direct Product and Semi-Direct Product of Groups, Fundamental Theorem (Structure Theorem) of Finite Abelian Groups, Cauchy's Theorem, Group Action, Sylow Theorems and their applications. Solvable Groups.

### 1.2.2 Ring and Fields

Ring, Integral Domain, Division Ring, Field. Ring Homomorphisms, Isomorphism Theorems. Ideals and Quotient Ring, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial and Power Series Rings. Divisibility Theory: Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss's Theorem, Eisenstein's criterion.

### 1.2.3 Books for Reference in Abstract Algebra

1. Dummit, D.S., Foote, R. M., Abstract Algebra, Second Edition, John Wiley and Sons, Inc.
2. Gallian, J., Contemporary Abstract Algebra, Narosa.
3. Roman, S., Fundamentals of Group Theory: An Advanced Approach, Birkhauser.
4. Malik, D. S., Mordesen, J. M., Sen, M. K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc.

5. Rotman, J., The Theory of Groups: An Introduction, Allyn and Bacon, Inc., Boston.
6. Rotman, J., A First Course In Abstract Algebra, Prentice Hall.
7. Pinter, Charles. C., A Book of Abstract Algebra, McGraw Hill.
8. Herstein, I. N., Topics in Abstract Algebra, Wiley Eastern Limited.
9. Fraleigh, J. B., A First Course in Abstract Algebra, Narosa.
10. Jacobson, N., Basic Algebra, I and II, Hindusthan Publishing Corporation, India.
11. Hungerford, T. W., Algebra, Springer.
12. Artin, M., Algebra, Prentice Hall of India.
13. Goldhaber, J. K., Ehrlich, G., Algebra, The Macmillan Company, Collier-Macmillan Limited, London.
14. Gopalakrishnan, N. S., University Algebra, New Age International.

### **1.3 C. P.- 3. Real Analysis: Marks 20(I.A.)+55(S.E.) [Credit 6]**

#### **1.3.1 Riemann-Stieltjes integral**

Existence and basic properties, Conditions of integrability, Algebra of R-S integrable functions, Integration by parts, Reduction to a Riemann integral, Integration of a continuous function with respect to a step function.

#### **1.3.2 The Lebesgue measure**

Definition of the Lebesgue outer measure on the power set of  $\mathbb{R}$ , countable subadditivity, Carathodory's definition of the Lebesgue measure and basic properties. Measurability of an interval (finite or infinite), Countable additivity, Characterizations of measurable sets by open sets,  $G_\delta$  sets, closed sets and  $F_\sigma$  sets. Measurability of Borel sets, Existence of non-measurable sets.

#### **1.3.3 Measurable functions**

Definition on a measurable set in  $\mathbb{R}$  and basic properties, Simple functions, Sequences of measurable functions, Measurable functions as the limits of sequences of simple functions, Lusin's theorem on restricted continuity of measurable functions, Egoroff's theorem.

### 1.3.4 The Lebesgue integral

Integrals of non-negative simple functions, The integral of non-negative measurable functions on arbitrary measurable sets in  $\mathbb{R}$  using integrals of non-negative simple functions, Monotone convergence theorem and Fatous lemma, The integral of Measurable functions and basic properties, Absolute character of the integral, Dominated convergence theorem, Inclusion of the Riemann integral.

### 1.3.5 Differentiation and Integration

Dini Derivatives, Functions of Bounded Variation, Definition and basic properties, Differentiation of an Integral, Absolutely Continuous Functions, Definition and basic properties, Integral of the Derivative.

### 1.3.6 Books for Reference in Real Analysis

1. Walter Rudin, Principles of Mathematical analysis, Tata McGraw Hill.
2. Walter Rudin, Real and Complex Analysis, Tata McGraw-Hill Publishing Co. Ltd.
3. T. M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi.
4. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York.
5. A. J. White, Real Analysis: an introduction, Addison-Wesley Publishing Co., Inc.
6. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer.
7. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited, New Delhi.
8. G. de Barra, Measure Theory and Integration, New Age International (P) Limited, New Delhi.
9. H. L. Royden, Real analysis, Macmillan Pub. Co. Inc. 4th edn. N.Y.
10. . J. H. Williamson, Lebesgue Integration, Holt Rinehart and Winston, Inc. N.Y.
11. P. R. Halmos, Measure Theory, Van Nostrand, Princeton.
12. R. G. Bartle, The Elements of Integration, John Wiley and Sons, Inc. N.Y.
13. Serge Lang, Analysis I and II, Addison-Wesley Publishing Co., Inc.
14. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi.
15. B. K. Lahiri and K. C. Ray: Real Analysis, World Press.
16. I. P. Natanson: Theory of Integrals of a Real Variable (Vol. I and II).

## 1.4 C. P.- 4. Ordinary Differential Equations and Special Functions:

Marks 20(I.A.)+55(S.E.) [Credit 6]

### 1.4.1 Ordinary Differential Equations

#### 1. Existence and Uniqueness:

First order ODE, Initial value problems, Existence theorem, Uniqueness, basic theorems. Ascoli Arzela theorem (statement only), Theorem on convergence of solution of initial value problems, Picard Lindelof theorem (statement only), Peano's existence theorem (statement only) and corollaries.

#### 2. Higher Order Linear ODE:

Higher order linear ODE, fundamental solutions, Wronskian, variation of parameters.

#### 3. Boundary Value Problems for Second Order Equations:

Ordinary Differential Equations of the Sturm-Liouville type and their properties, Application to Boundary Value Problems, Eigenvalues and Eigenfunctions, Orthogonality theorem, Expansion theorem. Green's function for Ordinary Differential Equations, Application to Boundary Value Problems.

### 1.4.2 Special Functions

#### 1. Singularities:

Fundamental System of Integrals, Singularity of a Linear Differential Equation. Solution in the neighbourhood of a singularity, Regular Integral, Equation of Fuchsian type, Series solution by Frobenius method.

#### 2. Hypergeometric Equation:

Hypergeometric Functions, Series Solution near zero, one, and infinity. Integral Formula, Differentiation of Hypergeometric Function.

#### 3. Legendre Equation:

Legendre Functions, Generating Function, Legendre Functions of First and Second kind, Laplace Integral, Orthogonal Properties of Legendre Polynomials, Rodrigue's Formula.

#### 4. Bessel Equation:

Bessel's Functions, Series Solution, Generating Function, Integral Representation of Bessel's Functions, Hankel Functions, Recurrence Relations, Asymptotic Expansion of Bessel Functions.

### 1.4.3 Books for Reference in Ordinary Differential Equations and Special Functions

1. Simmons, G. F., Differential Equations, Tata McGraw Hill.

2. Agarwal, Ravi P. and O' Regan D., An Introduction to Ordinary Differential Equations, Springer.
3. Coddington, E. A and Levinson, N., Theory of Ordinary Differential Equation, McGraw Hill.
4. Ince, E. L., Ordinary Differential Equation, Dover.
5. Estham, M. S. P., Theory of Ordinary Differential Equations, Van Nostrand Reinhold Compa.NY.
6. Piaggio, H. T. H., An Elementary Treatise On Differential Equations And Their Applications, G. Bell And Sons, Ltd.
7. Hartman, P., Ordinary Differential Equations, SIAM.
8. Zill, D. G., Cullen, M. R., Differential Equations with Boundary Value Problems, Brooks/Cole.

**1.5 IDC.- 1. Mathematical Logic, Integral Methods, Differential Equations and Vector Algebra:  
Marks 25(I.A.)+75(S.E.) [Credit 8] [For the students of the other departments]**

**1.5.1 Mathematical Logic [Marks 25]**

1. Simple and compound statements/propositions.
2. Logical connectives: negation, conjunction, disjunction, implication, equivalence.
3. Truth tables, tautology, logical equivalence, contradiction.
4. The algebra of propositions.

**1.5.2 Integral Methods [Marks 25]**

Simple problems on definite integral as the limit of sum. Working knowledge of Beta and Gamma functions (convergence to be assumed) and their interrelation (without proof). Use of the result:

$$\Gamma(m)\Gamma(1-m) = \frac{\pi}{\sin m\pi}, \text{ where } 0 < m < 1.$$

Computation of the following integrals using Beta and Gamma functions (when they exist):

$$\int_0^{\frac{\pi}{2}} \sin^n x \, dx, \int_0^{\frac{\pi}{2}} \cos^n x \, dx, \int_0^{\frac{\pi}{2}} \tan^n x \, dx \text{ etc.}$$

Working knowledge of double and triple integrals.

### 1.5.3 Differential Equations [Marks 25]

1. **Basics of ordinary differential equations:** Significance of ordinary differential equation. Geometrical and physical considerations. Formation of differential equation by elimination of arbitrary constants. Meaning of the solution of ordinary differential equation. Concepts of linear and non-linear differential equations.
2. **Equations of first order and first degree:** Existence theorem (statement only). Exact equation. Condition for exactness, Integrating factor. Rules of finding integrating factors (statements of relevant results only). Equations reducible to first order linear equations.
3. **Equations of first order but not of first degree:** Clairaut's equation. Singular solution.
4. **Applications:** Geometric applications, Orthogonal trajectories.
5. **Higher order linear equations with constant coefficients:** Complementary function. Particular Integral. Method of undetermined coefficients. Symbolic operator  $D$ . Method of variation of parameters. Euler's homogeneous equation and reduction to an equation of constant coefficients.

### 1.5.4 Vector Algebra [Marks 25]

1. Conditions for collinearity of three points and coplanarity of four points. Rectangular components of a Vector in two and three dimensions. Scalar and Vector products and triple products. Product of four vectors. Direct applications of Vector Algebra in
  - (i) Geometrical and Trigonometrical Problems,
  - (ii) Problems of Mechanics (Work done by a force, Moment of a force about a point).
2. Vector equations of straight lines and planes. Volume of a tetrahedron. Shortest distance between two skew lines.

### 1.5.5 Books for Reference in Mathematical Logic, Integral Methods, Differential Equations and Vector Algebra

#### Mathematical Logic

1. Introduction to Graph Theory: Douglas B. West (Prentice Hall of India)
2. Discrete Mathematics ( with Graph Theory ): E. G. Goodaire and M. M. Permenter (Prentice Hall of India)
3. Discrete Mathematics: J. K. Sharma (Macmillan)
4. Selected Topics on Discrete Mathematics: S. Kar (U. N. Dhur and Sons)

### **Integral Methods**

1. Application of Calculus: Sunil Kr. Maity and Sitansu Bandyopadhyay (Academic Publishers)
2. Application of Calculus: Debasish Sengupta (Books and Allied)
3. Calculus and its Applications: Goldstein, Lay, Schneider, Asmar (Pearson Education)
4. Integral Calculus: Shanti Narayan (S. Chand and Co.)
5. Integral Calculus - Differential Equations: B. C. Das and B. N. Mukherjee (U. N. Dhur and Sons)
6. An Introduction to Analysis- Integral Calculus: R. K. Ghosh and K. C. Maity (New Central Book Agency)
7. Integral Calculus and Differential Equations: Dipak Chatterjee (Tata McGraw Hill)

### **Differential Equations**

1. Integral Calculus and Differential Equations: Dipak Chatterjee (Tata McGraw Hill)
2. Differential Equations: Chakravorty, J. G. and Ghosh, P. R. (U. N. Dhur and Sons)
3. An Introduction to Differential Equations: R. K. Ghosh and K. C. Maity (New Central Book Agency)
4. Differential Equation and Laplace Transform: Das, A. N. (New Central Book Agency)
5. Differential Equations: G. F. Simmons (Tata McGraw Hill)

### **Vector Algebra**

1. Analytical Geometry and Vector Algebra: N. Datta and R. N. Jana (Shree-dhar Prakashani)
2. Analytical Geometry of two and three Dimensions and Vector Analysis: R. M. Khan (New Central Book Agency)
3. Vector Analysis: Chakravorty, J. G. and Ghosh, P. R. (U. N. Dhur and Sons)
4. Vector Analysis. Introduction to Tensor Analysis: Das, A. N. (U. N. Dhur and Sons)
5. Vector Analysis and An Introduction to Tensor Analysis: M. R. Spiegel (McGraw Hill)
6. Vector Analysis: R. K. Ghosh and K. C. Maity (New Central Book Agency)



# **SEMESTER 2**

## Chapter 2

# SEMESTER 2

### 2.1 C. P.- 5. Calculus of Several Variables: Marks 20(I.A.)+55(S.E.) [Credit 6]

$\mathbb{R}^n$  as a normed linear space, and  $L(\mathbb{R}^n, \mathbb{R}^m)$  as a normed linear space. Limits and continuity of functions from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ . The derivative at a point of a function from  $\mathbb{R}^n$  to  $\mathbb{R}^m$  as a linear transformation. The tangent space and linear approximation. The chain rule.

Partial derivatives, higher order partial derivatives and their continuity. Sufficient conditions for differentiability. Comparison between the differentiability of a function from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  and from  $\mathbb{C}$  to  $\mathbb{C}$ .

Examples of discontinuous and non-differentiable functions whose partial derivatives exist.

$C^1$  maps. Euler's theorem. Sufficient condition for equality of mixed partial derivatives. Proofs of the Inverse Function Theorem, the Implicit Function Theorem, and the Rank Theorem. Jacobians. The Hessian and the real quadratic form associated with it. Extrema of real-valued functions of several variables. Proof of the necessity of the Lagrange multiplier condition for constrained extrema.

Riemann Integral of real-valued functions on Euclidean spaces, measure zero sets, Fubini's Theorem. Partition of unity, change of variables. Stokes's Theorem and Divergence Theorem for integrals.

#### 2.1.1 Books for Reference in Calculus of Several Variables

1. Spivak, M., Calculus on Manifolds: A Modern Approach To Classical Theorems of Advanced Calculus, Addison-Wesley.
2. Munkres, J. R., Analysis on Manifolds, Addison-Wesley.
3. Apostol, T. M., Mathematical Analysis, Narosa Publishing House.
4. Apostol, T. M., Calculus Vol I and II, John Wiley and sons.
5. Fleming, W., Functions of Several Variables, 2nd Edition, Springer-Verlag.
6. Kaplan, W., Advanced Calculus, Pearson.

7. Ghorpade, S. R. and Limaye, B. V., A Course in Multivariable Calculus and Analysis, Springer.

## **2.2 C. P.- 6. Topology:**

**Marks 20(I.A.)+55(S.E.) [Credit 6]**

### **2.2.1 Topological Spaces and Continuous Functions**

Topology on a set, Examples of Topologies (Topological Spaces): Discrete Topology, Indiscrete Topology, Finite Complement Topology, Countable Complement Topology, Topologies on the Real Line:  $\mathbb{R}_l$ ,  $\mathbb{R}_K$ ,  $\mathbb{R}$  with usual Topology, etc., Finer and Coarser Topologies, Basis and Sub basis for a topology. Product topology on  $X \times Y$ , Subspace Topology.

Interior Points, Limit Points, Derived Set, Boundary of a set, Closed Sets, Closure and Interior of a set, Kuratowski closure operator and the generated topology.

Continuous Functions, Rules for Constructing Continuous Functions: Inclusion Map, Composition, by restricting the domain, by restricting/expanding the range, Pasting Lemma, Open maps, Closed maps and Homeomorphisms, Embedding of a Topological Space into another Topological Space.

(Infinite) Product Topology, Sub basis for product Topology defined by Projection Maps, Box Topology, Metric Topology.

### **2.2.2 Connectedness and Compactness**

Connected and Path Connected Spaces: Definitions, Examples and its simple properties, Connected subsets of the real line, Introduction to Components and Path Components, Local Connectedness.

Compact Spaces, Compact subsets of the real line, Heine-Borel Theorem.

### **2.2.3 Books for Reference in Topology**

1. Munkres, J. R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi.
2. Simmons, G. F., Introduction to Topology and Modern Analysis, McGraw-Hill.
3. Kumaresan, S., Topology of Metric Spaces, Narosa Publishing House.
4. Kelley, J. L., General Topology, Van Nostrand Reinhold Co., New York.
5. Young, J. G., Topology, Addison-Wesley Reading.
6. Willard, S., General Topology, Dover.
7. Dugundji, J., Topology, Allyn and Bacon.

8. Sierpinski, W., Introduction to General Topology, The University of Toronto Press, Canada.
9. Kuratowski, K., General Topology, Vol. I, Academic Press, New York and London.

## **2.3 C. P.- 7. Complex Analysis: Marks 20(I.A.)+55(S.E.) [Credit 6]**

### **2.3.1 Complex Differentiation**

Derivative of a complex function, Cauchy-Riemann equations, Necessary and sufficient criterion for complex differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates.

### **2.3.2 Complex Functions and Conformality**

Polynomial functions, Rational functions, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Conformal maps, Mobius Transformations.

### **2.3.3 Complex Integration**

The complex integral (over piecewise  $\mathcal{C}^1$  curves), Cauchy's fundamental theorem (statement only) and its consequences. Cauchy's integral formula. Derivative of an analytic function, Morera's theorem, Cauchy's inequality, Liouville's theorem, Fundamental theorem of classical algebra. Uniformly convergent series of analytic functions. Weierstrass Convergence Theorem. Power series. Taylor's theorem. Laurent's theorem.

### **2.3.4 Singularities**

Definitions and Classification of singularities of complex functions, Isolated singularities, Poles etc. Zeros of an analytic function. Limit points of zeros and poles. Identity Theorem. Riemann's theorem. Weierstrass-Casorati theorem. Theory of residues and its applications to contour integrals. Behaviour of a function at the point at infinity. Argument principle. Maximum modulus theorem and its applications. Rouché's theorem and its applications. Schwarz lemma. Idea of winding number.

### **2.3.5 Books for Reference in Complex Analysis**

1. Conway, J. B., Functions of One Complex Variable, Second Edition, Narosa Publishing House.
2. Marsden, J. E. and Hoffman, M. J., Basic Complex Analysis, Third Edition, W. H. Freeman and Company, New York.

3. Sarason, D., Complex Function Theory, Hindustan Book Agency, Delhi.
4. Ahlfors, L. V., Complex Analysis, McGraw-Hill.
5. Rudin, W., Real and Complex Analysis, McGraw- Hill Book Co.
6. Hille, E., Analytic Function Theory (2 vols.), Gonn and Co.
7. Ponnusamy, S., Foundations of Complex Analysis, Narosa.
8. A. I. Markushevich: Theory of Functions of a Complex Variable( Vol. I, II and III).
9. R. V. Churchill and J. W. Brown: Complex Variables and Applications.
10. E. C. Titchmarsh: The Theory of Functions.
11. E. T. Copson: An Introduction to the Theory of Functions of a Complex Variable.
12. H. S. Kasana: Complex Variables Theory and Applications.
13. J. M. Howie: Complex Analysis.
14. S. Narayan and P. K. Mittal: Theory of Functions of a Complex Variable.

## **2.4 C. P.- 8. Partial Differential Equations: Marks 20(I.A.)+55(S.E.) [Credit 6]**

### **2.4.1 First Order P.D.E.**

Formation and solution of PDE, Integral surfaces, Cauchy Problem order equation, Orthogonal surfaces, First order non-linear PDE, Characteristics, Compatible system, Charpit's method. Classification and canonical forms of PDE.

### **2.4.2 Second Order Linear P.D.E.**

Classification, reduction to normal form, Solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables.

### **2.4.3 Elliptic Differential Equations**

Derivation of Laplace and Poisson equation, Boundary Value Problem, Separation of Variables, Dirichlet's Problem and Neumann Problem for a rectangle, Interior and Exterior Dirichlet's problems for a circle, Interior Neumann problem for a circle, Solution of Laplace equation in Cylindrical and spherical coordinates, Examples.

#### **2.4.4 Parabolic Differential Equations**

Formation and solution of Diffusion equation, Dirac Delta function, Separation of variables method, Solution of Diffusion Equation in Cylindrical and spherical coordinates, Examples.

#### **2.4.5 Hyperbolic Differential Equations**

Formation and solution of one-dimensional wave equation, canonical reduction, Initial Value Problem, D'Alembert's solution, Vibrating string, Forced Vibration, Initial Value Problem and Boundary Value Problem for two-dimensional wave equation, Periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems, vibration of circular membrane, Uniqueness of the solution for the wave equation, Duhamel's Principle, Examples.

#### **2.4.6 Green's Function**

Green's function for Laplace Equation, methods of Images, Eigen function Method, Green's function for the wave and Diffusion equations. Laplace Transform method: Solution of Diffusion and Wave equation by Laplace Transform.

#### **2.4.7 Books for Reference in Partial Differential Equations**

1. Sneddon, I. N., Elements of Partial Differential Equations, McGraw Hill.
2. Williams, W. E., Partial Differential Equations.
3. Miller, F. H., Partial Differential Equations.
4. Petrovsky, I. G., Lectures on Partial Differential Equations.
5. Courant and Hilbert, Methods of Mathematical Physics, Vol-II.
6. Rao, K. S., Introduction to Partial Differential Equations, Prentice Hall.

### **2.5 IDC.- 2. Discrete Mathematics, Boolean Algebra, Graph Theory and Vector Analysis: Marks 25(I.A.)+75(S.E.) [Credit 8] [For the students of the other departments]**

#### **2.5.1 Discrete Mathematics [Marks 25]**

1. Principle of inclusion and exclusion. Pigeon-hole principle. Finite combinatorics. Generating functions. Partitions. Recurrence relations. Linear difference equations with constant coefficients.
2. Partial and linear orderings. Chains and antichains. Lattices. Distributive lattices. Complementation.

### 2.5.2 Boolean Algebra [Marks 25]

Huntington postulates for Boolean Algebra. Algebra of sets and Switching Algebra as examples of Boolean Algebra. Duality. Boolean functions. Normal forms. Karnaugh maps. Design of simple switching circuits.

### 2.5.3 Graph Theory [Marks 25]

1. **Graphs:** Undirected graphs. Directed graphs. Basic properties. Walk, Path, Cycle, Trail. Connected graphs. Components of a graph. Complete graph. Complement of a graph. Bipartite graphs. Necessary and sufficient condition for a Bipartite graph.
2. **Euler graphs:** Necessary and sufficient condition for a graph to be Euler graph. Königsberg Bridge Problem.
3. **Planar graphs:** Face-size equation, Euler's formula for a planar graph. To show: the graphs  $K_5$  and  $K_{3,3}$  are non-planar.
4. **Tree:** Basic properties. Spanning tree. Minimal Spanning tree. Kruskal's algorithm. Prim's Algorithm. Rooted tree. Binary tree.

### 2.5.4 Vector Analysis [Marks 25]

1. Vector differentiation with respect to a scalar variable: Vector functions of one scalar variable. Derivative of a vector. Second derivative of a vector. Derivatives of sums and products. Velocity and Acceleration as derivatives.
2. Elements of Differential Geometry: Curves in space. Tangent to a curve at a point, Normal plane, Serret-Frenet formulae, Principal Normal and Binormal, Osculating plane, Rectifying plane, Darboux vector, Twisted cubic.
3. Differential Operators: Concept of scalar and vector fields. Directional derivative. Gradient, Divergence, Curl and Laplacian.
4. Vector Integration: Line integrals as integrals of vectors, circulation, irrotational vector, work done by a vector. Conservative force, potential orientation. Statements (only) and verification of Green's theorem, Stoke's theorem and Divergence theorem.

### 2.5.5 Books for Reference in Discrete Mathematics, Boolean Algebra, Graph Theory and Vector Analysis

#### Discrete Mathematics, Graph Theory and Boolean Algebra

1. Introduction to Graph Theory: Douglas B. West (Prentice Hall of India)
2. Discrete Mathematics ( with Graph Theory ): E. G. Goodaire and M. M. Permenter (Prentice Hall of India)
3. Discrete Mathematics: J. K. Sharma (Macmillan)

4. Selected Topics on Discrete Mathematics: S. Kar (U. N. Dhur and Sons)
5. Higher Algebra- Abstract and Linear: S. K. Mapa (Sarat Book House)

### **Vector Analysis**

1. Analytical Geometry and Vector Algebra: N. Datta and R. N. Jana (Shree-dhar Prakashani)
2. Analytical Geometry of two and three Dimensions and Vector Analysis: R. M. Khan (New Central Book Agency)
3. Vector Analysis: Chakravorty, J. G. and Ghosh, P. R. (U. N. Dhur and Sons)
4. Vector Analysis. Introduction to Tensor Analysis: Das, A. N. (U. N. Dhur and Sons)
5. Vector Analysis and An Introduction to Tensor Analysis: M. R. Spiegel (McGraw Hill)
6. Vector Analysis: R. K. Ghosh and K. C. Maity (New Central Book Agency)



# **SEMESTER 3**

## Chapter 3

# SEMESTER 3

**3.1 In Semester-3, the Department will offer a set of Optional papers (O.P.) from O.P.- 1 to O.P.- 4 among which the students have to choose two optional papers.**

The Optional Papers be offered to the students on the basis of availability of Teachers and within the Framed Syllabi of the Optional Papers.

**3.2 O. P.- 1. Functional Analysis:  
Marks 20(I.A.)+55(S.E.) [Credit 6]**

### **3.2.1 Banach Spaces**

Normed Linear Spaces and its properties, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Riesz Lemma. Bounded Linear Transformations. Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Necessary and sufficient conditions for Bounded (Continuous) and Unbounded (Discontinuous) Linear functionals in terms of their kernel. Hyperplane, Necessary and sufficient conditions for a subspace to be hyperplane. Applications of Hahn-Banach Theorem, Dual Space, Examples of Reflexive Banach Spaces.  $L^p$  -Spaces and their properties.

### **3.2.2 Hilbert Spaces**

Real Inner Product Spaces and its Complexification, Cauchy-Schwarz Inequality, Parallelogram law, Pythagorean Theorem, Bessel's Inequality, Gram-Schmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Orthogonal Complement and Projections. Riesz Representation Theorem for Hilbert Spaces, Adjoint of an Operator on a Hilbert Space with examples, Reflexivity of Hilbert Spaces, Definitions and

examples of Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators and Unitary Operators. Introduction to Spectral Properties of Bounded Linear Operators.

### 3.2.3 Books for Reference in Functional Analysis

1. Limaye, B. V., Functional Analysis, Wiley Eastern Ltd.
2. Kreyszig, E., Introductory Functional Analysis and its Applications, John Wiley and Sons, New York.
3. Brown, A. and Percy, C., Introduction to Operator Theory I: Elements of Functional Analysis, Springer-Verlag, New York.
4. Suhubi, E. S., Functional Analysis, Springer, New Delhi.
5. Aliprantis, C. D., Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.
6. Ponnusamy, S., Foundations of Functional Analysis, Narosa.
7. Goffman, C., Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi.
8. Bachman, G., Narici, L., Functional Analysis, Academic Press.
9. Taylor, A. E., Introduction to Functional Analysis, John Wiley and Sons, New York.
10. Simmons, G. F., Introduction to Topology and Modern Analysis, McGraw-Hill.
11. Conway, J. B., A Course in Functional Analysis, Springer Verlag, New York.
12. Rudin, W., Functional Analysis, Tata McGraw Hill.

### 3.3 O. P.- 2. Measure and Integration Theory: Marks 20(I.A.)+55(S.E.)[Credit 6]

Measure on a  $\sigma$ -algebra of sets.

Construction of measure by means of outer measure, regular outer measure and metric outer measure.

Integration on a measure space.

Signed measure space. Decomposition of signed measures. Integration on a signed measure space.

Absolute continuity of a signed measure relative to a positive measure.

Radon- Nikodyn Theorem.

Product measure: Fubinis Theorem and Tonellis Theorem.

### 3.3.1 Books for Reference in Measure and Integration Theory

1. S. K. Berberian, Measure and integration. Chelsea Publishing Company, N. Y.
2. P. R. Halmos, Measure Theory, Van Nostrand. Princeton.
3. H. L. Royden, Real analysis, Macmillan Publishing Co., Inc. 4th Edition.
4. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd., New Delhi.
5. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, Delhi.
6. J. H. Williamson, Lebesgue Integration, Holt Rinehart and Winston Inc., New York.
7. R. G. Bartle, The Elements of Integration, John Wiley and Sons, Inc., New York.
8. T. G. Hawkins, Lebesgue's Theory of Integration: Its Origins and Development, Chelsea, New York.
9. K. R. Parthasarathy, Introduction to Probability and Measure, Macmillan Co. India Ltd., Delhi.
10. Serge Lang, Analysis I and II, Addison-Wesley Publishing Co. Inc.

## 3.4 O. P.- 3. Graph Theory: Marks 20(I.A.)+55(S.E.) [Credit 6]

### 3.4.1 Fundamental Concepts

Basic Definitions. Graphs, Vertex degrees, Walks, Paths, Trails, Cycles, Circuits, Subgraphs, Induced subgraph, Cliques, Components, Adjacency Matrices, Incidence Matrices, Isomorphisms.

### 3.4.2 Graphs with special properties

Complete Graphs. Bipartite Graphs. Connected Graphs,  $k$ -connected Graphs, Edge-connectivity, Cut-vertices, Cut-edges. Eulerian Trails, Eulerian Circuits, Eulerian Graphs: characterization, Hamiltonian (Spanning) Cycles, Hamiltonian Graphs: Necessary condition, Sufficient conditions (Dirac, Ore, Chvatal, Chvatal-Erdos), Hamiltonian Closure, Travelling Salesman Problem.

### 3.4.3 Trees

Basic properties, distance, diameter. Rooted trees, Binary trees, Binary Search Trees. Cayleys Formula for counting number of trees. Spanning trees of a connected graph, Depth first search (DFS) and Breadth first search (BFS) Algorithms, Minimal spanning tree, Shortest path problem, Kruskals Algorithm, Prims Algorithm, Dijkstras Algorithm. Chinese Postman Problem.

### 3.4.4 Coloring of Graphs

Vertex coloring: proper coloring,  $k$ -colorable graphs, chromatic number, upper bounds, Cartesian product of graphs, Structure of  $k$ -chromatic graphs, Mycielskis Construction, Color-critical graphs, Chromatic Polynomial, Clique number, Independent (Stable) set of vertices, Independence number, Clique covering, Clique covering number. Perfect graphs: Chordal graphs, Interval graphs, Transitive Orientation, Comparability graphs. Edge-coloring, Edge-chromatic number, Line Graphs.

### 3.4.5 Books for Reference in Graph Theory

1. Douglas B. West, Introduction to Graph Theory, Prentice-Hall of India Pvt. Ltd., New Delhi.
2. F. Harary, Graph Theory, Addison-Wesley.
3. K. R. Parthasarathi, Basic Graph Theory, Tata McGraw-Hill Publ. Co. Ltd., New Delhi.
4. L. R. Foulds, Graph Theory Applications, Narosa Publishing House, New Delhi.
5. J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, Elsevier science.
6. G. Chartrand and L. Lesniak, Graphs and Digraphs, Chapman and Hall.
7. Narsingh Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice-Hall of India Pvt. Ltd., New Delhi.

## 3.5 O. P.- 4. Numerical Analysis: Marks 20(I.A.)+55(S.E.) [Credit 6]

### 3.5.1 Numerical Solution of System of Linear Equations

Triangular factorization methods, Matrix inversion method, Iterative methods- Jacobi method, Gauss Jacobi method, Gauss-Seidel method, Successive over relaxation (SOR) method and convergence condition of Iterative methods, Rate of convergence of methods.

### **3.5.2 Solution of Non-linear Equations**

Iteration methods: Tchebyshev method, Multipoint method, Modified Newton-Raphson method (for simple or repeated real roots), Rate of convergence of all iteration methods.

### **3.5.3 System of Non-linear Equations**

Newton's Method, Quasi-Newton's method.

### **3.5.4 Numerical Solution of Initial Value Problem for ODE**

First order Equation: Runge-Kutta methods, Multistep predictor-corrector methods, Convergence and stability.

### **3.5.5 Two Point Boundary Value Problem for ODE**

Finite difference method, Shooting Method.

### **3.5.6 Numerical Solution of PDE by Finite Difference Method**

Parabolic equation in one dimension (Heat equation), Explicit finite difference method, Implicit Crank-Nickolson method, Hyperbolic equation in one-space dimension (Wave equation)- Finite difference method, Convergence and Stability.

### **3.5.7 Books for Reference in Numerical Analysis**

1. Jain, M. K., Iyenger, S. R. K. and Jain, R. K., Numerical Methods for Scientific and Engineering Computation, New Age International.
2. Atkinson, K. E., An Introduction to Numerical Analysis, John Wiley and Sons.
3. Smith, G. D., Numerical Solution of Partial Differential Equations.
4. Berzin and Zhidnov, Computing methods.
5. Isacson and Keller, Analysis of Numerical methods.
6. Ralston and Rabinowitz, A First Course in Numerical Analysis.
7. Jain, M. K., Numerical Solution of Differential Equations.
8. Fox, L., Numerical Solution of Ordinary and Partial Differential Equations, Oxford Univ. Press.

**3.6** In Semester-3, the Department will offer a set of Special papers (S.P.) from S.P.- 1 to S.P.- 8 among which the students have to choose two special papers. Part I and II of the corresponding special papers will be taught in semester 3 and 4 respectively.

The Optional Papers be offered to the students on the basis of availability of Teachers and within the Framed Syllabi of the Optional Papers.

**3.7 S. P.- 1. Advanced Real Analysis- I:**  
**Marks 20(I.A.)+55(S.E.) [Credit 6]**

**3.7.1 Ordinal Numbers**

Order type, well ordered sets, transfinite induction, ordinal numbers, comparability of ordinal numbers, Arithmetic of ordinal numbers. First uncountable ordinal ( $\Omega$ ).

**3.7.2 Descriptive properties of sets**

Perfect sets, every closed set is the union of a perfect set and a finite or denumerable set. Nowhere dense set. First category, second category and residual sets. In a complete metric space  $X$  every subset of  $X$  is residual in  $X$  if and only if it contains a dense  $G_\delta$ -set. Borel sets of order type  $\alpha (< \Omega)$  and its properties.

**3.7.3 Functions of special classes**

Baire class functions of order type  $\alpha (< \Omega)$  and its properties. Relation Between Baire functions and Borel sets.

**3.7.4 Continuity**

Lower and upper semi-continuous functions with their properties. Absolute continuity and Lusin ( $N$ ) condition. Lebesgue density point of a set and Lebesgue density theorem, Approximate continuity and its simple properties.

**3.7.5 Derivative**

The Vitali-covering theorem, Dini's derivatives and its properties. Derivative of a monotone function, Determining a function by its derivative. Lebesgue point.

**3.7.6 Books for Reference in Advanced Real Analysis - I**

1. Bruckner, A., Bruckner, J. B. and Thomson, J. B., Real Analysis.

2. Goffman, C., Real Functions.
3. Jeffrey, R. L., The Theory of Functions of a Real Variable.
4. Natanson, I. P., Theory of Functions of a Real Variable, Vol. I and II.
5. Hobson, E. W., Theory of Functions of a Real Variable, Vol. I and II.
6. Royden, H. L., Real Analysis.
7. Munroe, M. E., Introduction to Measure and Integration.
8. Lee, P. Y., Lanzhou Lectures on Henstock Integration.
9. Das, A. G., Generalized Riemann Integral.

### **3.8 S. P.- 2. Advanced Complex Analysis - I: Marks 20(I.A.)+55(S.E.) [Credit 6]**

The Functions  $M(r)$ ,  $A(r)$ , Hadamard Theorem on Growth of  $\log M(r)$ , Schwarz Inequality, Borel-Caratheodory Inequality. Open mapping theorem.

Entire Functions, Growth of an Entire Function, Order and Type and their Representations in terms of the Taylor Coefficients, Distribution of Zeros, Schottky's Theorem (no proof), Picard's Little Theorem, Weierstrass Factor Theorem, The Exponent of Convergence of Zeros, Hadamard Factorization Theorem, Canonical Product, Borel's First Theorem, Borel's Second Theorem (statement only).

Analytic Continuation, uniqueness, continuation by the method of power series, Natural Boundary, Existence of singularity on the circle of convergence, Analytic Element, Global Analytic Function, Concept of Analytic Manifolds, Multiple Valued Conditions, Branch Points and Branch Cut, Riemann Surfaces for the functions  $\sqrt{z}$ ,  $\log z$ .

#### **3.8.1 Books for Reference in Advanced Complex Analysis - I**

1. E. T. Copson: An Introduction to the Theory of Functions of a Complex Variable.
2. E. C. Titchmarsh: The Theory of Functions.
3. A. I. Markushevich: Theory of Functions of a Complex Variable (Vol. I, II and III).
4. L. V. Ahlfors: Complex Analysis.
5. J. B. Conway: Functions of One Complex Variable.
6. A. I. Markushevich: The Theory of Analytic Functions, A Brief Course.
7. G. Valiron: Integral Functions.
8. C. Caratheodory: Theory of Functions of a Complex Variable.
9. R. P. Boas: Entire Functions.



### **3.9 S. P.- 3. Advanced Topology - I: Marks 20(I.A.)+55(S.E.) [Credit 6]**

Compactness, Limit point compactness, sequentially compact spaces, countably compact spaces. Locally compact spaces.

Countability Axioms, The Separation Axioms, Lindelof spaces, Regular spaces, Normal spaces, Urysohn Lemma, Tietze Extension Theorem.

Tychonoff Theorem and Compactification: Tychonoff Theorem, Completely Regular spaces, Local Compactness, One-point compactification, Stone-Cech Compactification. Metrization: Urysohn Metrization Theorem, Topological Embedding, Embedding Theorem of a regular space with countable base in  $\mathbb{R}^n$ , Partitions of Unity, Topological  $m$ - Manifolds, Embedding Theorem of a compact  $m$ -manifold in  $\mathbb{R}^n$ .

Local Finiteness, Nagata-Smirnov Metrization Theorem, Paracompactness, Stone's Theorem, Local Metrizable, Smirnov Metrization Theorem. Uniform Spaces.

#### **3.9.1 Books for Reference in Advanced Topology - I**

1. Munkres, J. R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi.
2. Dugundji, J., Topology, Allyn and Bacon.
3. Simmons, G. F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
4. Kelley, J. L., General Topology, Van Nostrand Reinhold Co., New York.
5. Bourbaki, N., Topologie Generale.
- 6 . Hocking, J. and Young, G., Topology, Addison-Wesley Reading.
6. Steen, L. and Seebach, J., Counter Examples in Topology, Holt, Reinhart and Winston, New York.

### **3.10 S. P.- 4. Advanced Functional Analysis - I: Marks 20(I.A.)+55(S.E.) [Credit 6]**

Normed linear spaces, Banach spaces. Stone-Weierstrass Theorem. Ascoli-Arzelà Theorem. Bounded linear operators. Hahn-Banach theorem. Dual of a normed linear space. Computing the duals of well known Banach spaces. The double dual. Reflexive spaces. Baire's Category Theorem. Uniform Boundedness Theorem. Strong Convergence and Weak Convergence of sequences. Convergence of sequence of operators. Linear Topological Spaces, Locally Convex Spaces and their Characterization in terms of a family of Seminorms.

### 3.10.1 Books for Reference in Advanced Functional Analysis - I

1. Rudin, W., Real and complex analysis, McGraw-Hill.
2. Rudin, W., Functional analysis, McGraw-Hill.
3. Conway, J. B., A course in functional analysis, GTM (96), Springer-Verlag.
4. Yosida, K., Functional analysis, Springer-Verlag.
5. Katznelson, Y., An introduction to harmonic analysis, Dover Publications.
6. Stein, E. M. and Shakrachi, R., Fourier Analysis: An Introduction, Princeton Lectures in Analysis.
7. Hernez, E. and Weiss, G., A first course on wavelets, Studies in Advanced Mathematics, CRC Press.
8. Kelley, J. L. and Namioka, I., Linear Topological Spaces, D.Van Nostrand Company.
9. Aliprantis, C. D., Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.
10. Goffman, C. and Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi.
11. Taylor, A. E., Introduction to Functional Analysis, John Wiley and Sons, New York.

### 3.11 S. P.- 5. Operations Research - I: Marks 20(I.A.)+55(S.E.) [Credit 6]

#### 3.11.1 Allocation Problems

- (i) Transportation Problems  
Mathematical representation of transportation problems, Unbalanced transportation problems, Degenerate transportation problems, Resolution of degeneracy.
- (ii) Assignment Problems  
Mathematical representation of assignment problems, Reduction theorems, Solution methods of assignment problems, Hungarian method of zero assignment technique, Restriction assignment, Negative cost, etc., Variations of assignment problem, Multiple optimal solution, Maximization in assignment problem, Unbalanced assignment problem.
- (iii) Travelling Salesman Problem/ Routing Problem  
Origin of travelling salesman problem, Symmetrical and asymmetrical problems, Mathematical representation of problems, Solution techniques for such problems using zero assignment/unit assignment, etc.

### 3.11.2 Competitive Strategy (Game Theoretic Problems)

Introduction, Minimax/Maximin criterion, Rectangular games, Strategies, Pure and Mixed strategy problems, Saddle point, Graphical methods of solving  $2 \times n$  and  $m \times 2$  games, Dominance principle, Equivalence of rectangular games and L. P. P. solution by Simplex method.

### 3.11.3 Queueing Theory (Theory of Waiting Lines)

Introduction, Queueing system, Queue disciplines FIFO, FIFS, LIFO, SIRO, FILO etc. The Poisson process (Pure birth process), Arrival distribution theorem, Properties of Poisson process, Distribution of inter arrival times (exponential process), Markovian property of inter arrival times, Pure death process (Distribution of departures), Derivation of service time distribution, Analogy of exponential service times with Poisson arrivals, Erlang service time distribution, Kendals notations, Probabilistic queueing models ( $M = M = 1$ ): (1=FCFS), General Erlang model, (M=M=1) : (N=FCFS); (M=M=S) : (1=FCFS) and their properties.

### 3.11.4 Books for Reference in Operations Research - I

1. S. D. Sharma, Operations Research.
2. Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.
3. Sasieni Maurice, Arther Yaspan, Lawrence Friedman, OR methods and Problems.
4. H. S. Taha, Operations Research.
5. T. L. Satty, Operations Resarch.

## 3.12 S. P.- 6. Fluid Mechanics - I: Marks 20(I.A.)+55(S.E.) [Credit 6]

### 3.12.1 Viscous incompressible fluid flow

Field equations, Boundary conditions, Reynold's number, Vorticity equation, Circulation, Flow through parallel plates, Flow through pipes of circular and elliptic cross sections.

### 3.12.2 Inviscid Compressible Fluid

Field equations, Circulation, Propagation of small disturbance. Mach number and cone, Bernoulli's equation. Irrotational motion, Velocity potential. Bernoulli's equation in terms of Mach number. Pressure, density, temperature in terms of Mach number, Critical conditions. Steady channel flow, Area-velocity relation. Mass flow through a converging nozzle. Flow through a de-Laval nozzle. Normal shock waves, Governing equations and the solution. Entropy change.

### 3.12.3 Vortex Motion

Vortex line, Vortex tube, Properties of the vortex, Strength of the vortex, Rectilinear vortices, Velocity component, centre of vortices. A case of two vortex filaments, vortex pair. Vortex doublet. Image of vortex filament with respect to a plane. An infinite single row of parallel rectilinear vortices of same strength. Two infinite row of parallel rectilinear vortices, Karman's vortex street. Rectilinear vortex with circular section. Rankine's combine vortex. Rectilinear vortices with elliptic section.

### 3.12.4 Books for Reference in Fluid Mechanics - I

1. Prandtl, L., Essential of fluid dynamics, Springer.
2. White, F. M., Viscous Fluid Flow, McGraw Hill.
3. Panton, R. L., Incompressible Flow, John Wiley and Sons.
4. Rosenhead, L., Laminar Boundary Layer, Dover.
5. Sherman, F. S., Viscous Flow (McGraw Hill).
6. Pai, S. I., Viscous Flow Theory, D. Van Nostrand.
7. Schlichting, H., Boundary Layer Theory, Springer.
8. Chorlton, F., Text Book of Fluid Dynamics, CBS Pub.
9. Love, A. E., A treatise on mathematical theory of elasticity, McGraw Hill Book Co.
10. Kondepudi, D. and Prigogine, I., Modern thermodynamics, John Wiley and Sons, Inc.
11. Landau, L. M. and Lifshitz, E. M., Fluid Mechanics, Butterworth Heine-  
mann.

## 3.13 S. P.- 7. Nonlinear Differential Equations and Dynamical Systems - I: Marks 20(I.A.)+55(S.E.) [Credit 6]

### 3.13.1 Nonlinear Differential Equations

Autonomous systems, Flows, Phase space, existence and uniqueness of solutions, stability, Lyapunov function, fixed points, saddle, nodes, focus, stable, unstable and centre subspaces, Hartmann-Grabmann Theorem (statement only), Examples, Linearization, geometrical properties, averaging methods, perturbation method, method of multiscales, forced oscillations, Poincare maps, periodic orbits, invariant sets, limit sets, attracting and repelling sets, Poincare Bendixson theorem (statement only), bifurcations, simple examples, Hopf bifurcation.

### **3.13.2 Books for Reference in Nonlinear Differential Equations and Dynamical Systems - I**

1. D. W. Jordan and P. Smith, Nonlinear Ordinary Differential Equations, OUP.
2. E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw Hill.
3. R. L. Devaney, An Introduction to Chaotic Dynamical Systems, Westview Press, 2003.
4. B. Hasselblatt and A. Katok, A first Course in Dynamics, CUP.
5. R. A. Holmgren, A first course in discrete dynamical systems, Springer.
6. Alligood, Sauer and York, Chaos, an introduction to dynamical systems, Springer.
7. K. Falconer, Foundation to fractal geometry, CUP.

### **3.14 S. P.- 8. Theory of Relativity, Astrophysics and Cosmology - I: Marks 20(I.A.)+55(S.E.) [Credit 6]**

#### **3.14.1 Theory of Relativity and Astrophysics**

Transformation of coordinates. Tensors. Algebra of Tensors. Symmetric and skew symmetric Tensors. Contraction of tensors and quotient law. Orthonormal bases. Riemannian metric. Parallel transport. Christoffel Symbols. Covariant derivatives. Intrinsic derivatives and geodesics, Riemann Christoffel curvature tensor and its symmetry properties. Bianchi identities and Einstein tensor. Energy momentum tensor of an electromagnetic field. Einstein-Maxwell equations. Reissner-Nordstrom solution.

Review of the special theory of relativity and the Newtonian Theory of gravitation. Principle of equivalence and general covariance, geodesic principle. Newtonian approximation of relativistic equations of motion. Einstein's field equations and its Newtonian approximation.

Schwarzschild external solution and its isotropic form. Planetary orbits and analogues of Kepler's Laws in general relativity. Advance of perihelion of a planet. Bending of light rays in a gravitational field. Gravitational redshift of spectral lines. Radar echo delay. Rotating Kerr solution and geodesics in it. Gravitational Collapse, White dwarf and Neutron stars, Pulsars, Black holes, Wormholes.

#### **3.14.2 Books for Reference in Theory of Relativity, Astrophysics and Cosmology - I**

1. W. M. Smart, Text book of Spherical Astronomy.

2. K. D. Abhyankar, Astrophysics: Stars and Galaxies, Tata McGraw Hill Publication.
3. Martin V. Zombeck, Handbook of Space Astronomy and Astrophysics, Cambridge University Press.
4. J. V. Narlikar, The Structure of the Universe, Prentice Hall.
5. T. Padmanabhan, An Invitation to Astrophysics, World Scientific.
6. C. E. Weatherburn, An Introduction to Riemannian Geometry and the tensor Calculus, Cambridge University Press.
7. H. Stephani, General Relativity: An Introduction to the theory of the gravitational field, Cambridge University Press.
8. A. S. Eddington, The Mathematical Theory of Relativity, Cambridge University Press.
9. J. V. Narlikar, General Relativity and Cosmology, The Macmillan Company of India Limited.
10. R. Adlev, M. Bazin, M. Schiffer, Introduction to general relativity, McGraw Hill Inc.
11. B. F. Shutz, A first course in general relativity, Cambridge University Press.
12. S. Weinberg, Gravitation and Cosmology: Principles and applications of the general theory of relativity, John Willey and Sons Inc.
13. R. K. Sachs and H. Wu., General Relativity for Mathematician, Springer Verlag.
14. L. D. Landau and E. M. Lifshitz, The classical theory of Fields, Pergamon Press.
15. S. K. Bose, Introduction to general relativity, Wiley Eastern, New Delhi.
16. N. Duric, Advanced Astrophysics, Cambridge University Press.
17. W. Kundt, Astrophysics: A new approach, Springer.
18. P. Lena, F. Lebrun and F. Mignard, Observational Astrophysics, Springer.
19. N. Straumann, General Relativity and Relativistic Astrophysics, Springer-Verlog.
20. T. Padmanabhan, Theoretical Astrophysics (Vol. I, II and III), Cambridge University Press.

**3.15 Grand Viva, Seminar and group discussion:  
Marks 50+25+25 [Credit 4+2+2=8]**

**Grand Viva and the Seminar will have to be given by the students on the basis of subject offered to the students in 3rd semistar.**

# **SEMESTER 4**

## Chapter 4

# SEMESTER 4

**4.1 In Semester-4, the Department will offer a set of Optional papers (O.P.) from O.P.- 5 to O.P.- 8 among which the students have to choose two optional papers.**

The Optional Papers be offered to the students on the basis of availability of Teachers and within the Framed Syllabi of the Optional Papers.

**4.2 O. P.- 5. Differential Geometry:  
Marks 20(I.A.)+55(S.E.) [Credit 6]**

### **4.2.1 Tensors**

Tensor and their transformation laws, Tensor algebra, Contraction, Quotient law, Reciprocal tensors, Kronecker delta, Symmetric and skew-symmetric tensors, Metric tensor, Riemannian space, Christoffel symbols and their transformation laws, Covariant differentiation of a tensor, Riemannian curvature tensor and its properties, Bianchi identities, Ricci-tensor, Scalar curvature, Einstein space.

### **4.2.2 Curves in Space**

Parametric representation of curves, Helix, Curvilinear coordinates in  $E_3$ . Tangent and first curvature vector, Frenet formulae for curves in space, Frenet formulae for curves in  $E_n$ . Intrinsic differentiation, Parallel vector fields, Geodesic.

### **4.2.3 Surfaces**

Parametric representation of a surface, Tangent and Normal vector field on a surface, The first and second fundamental tensor, Geodesic curvature of a surface curve, The third fundamental form, Gaussian curvature, Isometry of sur-



faces, Developable surfaces, Weingarten formula, Equation of Gauss and Codazzi, Principal curvature, Normal curvature, Meusniers theorem.

#### **4.2.4 Books for Reference in Differential Geometry**

1. I. S. Sokolnikoff, Tensor Calculus and Application to Geometry and Mechanics.
2. T. T. Wilmore, An Introduction to Differential Geometry.
3. Bary Spain, Differential Geometry.

### **4.3 O. P.- 6. Integral Equation and Integral Transform: Marks 20(I.A.)+55(S.E.) [Credit 6]**

#### **4.3.1 Integral Equation**

Basic concepts, Volterra integral equations, Relationship between linear differential equations and Volterra equations, Resolvent kernel, Method of successive approximations, Convolution type equations, Volterra equation of first kind, Abel's integral equation, Fredholm integral equations, Fredholm equations of the second kind, the method of Fredholm determinants, Iterated kernels, Integral equations with degenerate kernels, Eigen values and eigen functions of a Fredholm alternative, Construction of Green's function for Boundary Value Problems, Singular integral equations.

#### **4.3.2 Laplace Transform**

Laplace transform, properties of Laplace transform, inversion formula of Laplace transform (Bromwich formula), Convolution theorem, Application to ordinary and partial differential equations.

#### **4.3.3 Fourier Transform**

Properties of Fourier transform, Inversion formula, Convolution, Parseval relation, Multiple Fourier transform, Bessels inequality, Application of transform to Heat, Wave and Laplace equations.

#### **4.3.4 Hankel Transform**

Hankel transform, Inversion formula of Hankel transform, Parseval relation, Finite Hankel transform, Application to partial differential equations.

#### **4.3.5 Z-transform**

Definition and properties of Z-transform. Z-transforms of some standard functions. Inverse Z-transforms. Applications.

#### 4.3.6 Books for Reference in Integral Equation and Integral Transform

1. Courant and Hilbert, Methods of Mathematical Physics, Vol-I, II.
2. Sneddon, I. N., The Uses of Integral Transforms, McGraw Hill.
3. Sneddon, I. N., Fourier Transform, Dover.
4. Lovitt, W. V., Linear Integral Equations, Dover.
5. Tricomi, F. G., Integral Equations, Dover.
6. Andrews, L. and Shivamoggi, V. K., Integral Transforms for Engineers, SPIE Press.
7. Debnath, L. and Bhatta, D., Integral Transforms and Their Applications, CRC Press.
8. Davics, B., Integral Transforms and Their Applications, Springer.
9. Pinkus, A. and Zafrany, S., Fourier Series and integral transforms, Cambridge University Press.

#### 4.4 O. P.- 7. Classical Mechanics: Marks 20(I.A.)+55(S.E.) [Credit 6]

Dynamical systems, Generalized coordinates, Degrees of freedom, Principle of virtual work. D'Alembert's principle. Unilateral and bilateral constraints. holonomic and nonholonomic system. Lagrange's equations for holonomic systems. Lagrange's equation for impulsive forces and for systems involving dissipative forces. Conservation theorems. Hamilton's principle and principle of least action. Hamilton's canonical equations. Canonical transformation with different generating functions. Lagrange and Poisson brackets and their properties. Hamilton-Jacobi equations and separation of variables. Routh's equations, Poisson's identity. Jacobi-Poisson Theorem. Brachistochrone problem. Configuration space and system point. Variation of functional, necessary and sufficient conditions for extrema, Euler-Lagrange equations and its applications: Geodesic, minimum surface of revolution, Brachistochrone problem and other boundary value problems in ordinary and partial differential equations.

##### 4.4.1 Books for Reference in Classical Mechanics

1. Goldstein, H., Classical Mechanics, Dover.
2. Arnold, V. I. (Vogtmann, K., Weinstein, A.), Mathematical Methods of Classical Mechanics, Springer (GTM).
3. Rana, N. C. and Jog, P. S., Classical Mechanics, Tata McGraw Hill.
4. Louis, N. H. and Finch, J. D., Analytical Mechanics.

5. Ramsay, A. S., Dynamics, Part-II.

#### **4.5 O. P.- 8 Combinatorial Mathematics: Marks 20(I.A.)+55(S.E.) [Credit 6]**

General Principles of Enumeration, Counting of Sub-Sets, Partitions, Binomial Theorem, Multinomial Theorem.

Principles of Inclusion and Exclusion, Derangements, Rook polynomials, Arrangement with Forbidden Positions.

General Principles of Enumeration, Counting.

Latin Square, Quasi-group, Orthogonal Latin Square.

##### **4.5.1 Books for Reference in Combinatorial Mathematics**

1. Peter J. Cameron, Combinatorics: Topics, Techniques, Algorithms, Cambridge University Press.
2. J. H. Lint and R. M. Wilson, A Course in Combinatorics, Cambridge University Press.
3. Ralph P. Grimaldi, Discrete and Combinatorial Mathematics, AWL.
4. Joe L. Mott, Abraham Kandel and Theodore P. Baker, Discrete Mathematics for Computer Scientists and Mathematicians (Prentice-Hall India).

#### **4.6 S. P.- 1. Advanced Real Analysis - II: Marks 20(I.A.)+55(S.E.) [Credit 6]**

##### **4.6.1 Fourier Series**

Trigonometric series, Fourier series, Dirichlet's kernel, pointwise convergence-Dini's test, Jordan test, convergence of Cesaro means-Fejer's theorem, Lebesgue-Fejer's theorem, Riemann's theorem. Cantor's uniqueness theorem.

##### **4.6.2 Integration on $\mathbb{R}$**

Henstock integral: Gauge functions, -finite partition, Cousin lemma, definition of Henstock integral and examples, Saks-Henstock lemma, Linearity property, Fundamental theorem. Relation of Henstock integral with Newton, Riemann and Lebesgue integrals. Absolute integrability of Henstock integral, Monotone and Dominated Convergence theorem of Henstock integral.

### 4.6.3 General Measure and Integration

Measure space, measurable functions, integration of non-negative function, convergence theorems, Fatou's lemma, Signed measure, positive and negative sets. Hahn and Jordan decomposition theorems. Absolute continuous and singular measures, Radon-Nikodym theorem and its consequences.

### 4.6.4 Books for Reference in Advanced Real Analysis-II

1. Bruckner, A., Bruckner, J. B. and Thomson, J. B., Real Analysis.
2. Goffman, C., Real Functions.
3. Jeffrey, R. L., The Theory of Functions of a Real Variable.
4. Natanson, I. P., Theory of Functions of a Real Variable, Vol. I and II.
5. Hobson, E. W., Theory of Functions of a Real Variable, Vol. I and II.
6. Royden, H. L., Real Analysis.
7. Munroe, M. E., Introduction to Measure and Integration.
8. Lee, P. Y., Lanzhou Lectures on Henstock Integration.
9. Das, A. G., Generalized Riemann Integral.

## 4.7 S. P.- 2. Advanced Complex Analysis - II: Marks 20(I.A.)+55(S.E.) [Credit 6]

Harmonic Functions, Characterization of Harmonic Functions by Mean-Value Property, Poisson's Integral Formula, Dirichlet Problem for a Disc. Doubly Periodic Functions, Weierstrass Elliptic Functions. Meromorphic Functions, Expansions, Definition of the functions  $m(r, a)$ ;  $N(r, a)$  and  $T(r)$ . Nevanlinna's First Fundamental Theorem, Cartan's Identity and Convexity Theorems, Order of Growth, Order of a Meromorphic Function, Comparative Growth of  $\log M(r)$  and  $T(r)$ , Nevanlinna's Second Fundamental Theorem, Estimation of  $S(r)$  (statement only), Nevanlinna's Theory of Deficient Values, Upper Bound of the Sum of Deficiencies. Nevanlinna's five-point uniqueness theorem. Milloux theorem.

### 4.7.1 Books for Reference in Advanced Complex Analysis - II

1. E. C. Titchmarsh, The Theory of Functions.
2. E. T. Copson, An Introduction to the Theory of Functions of a Complex Variable.
3. A. I. Markushevich, Theory of Functions of a Complex Variable, (Vol. I, II, III).
4. W. Kaplan, An Introduction to Analytic Functions.

5. H. Cartan, Theory of Analytic Functions.
6. W. K. Hayman, Meromorphic Functions.
7. L. Yang, Value Distribution Theory.
8. C. C. Yang and H. X. Yi, Uniqueness Theory of meromorphic functions.

#### **4.8 S. P.- 3. Advanced Topology - II: Marks 20(I.A.)+55(S.E.) [Credit 6]**

Nets and Filters: Directed Sets, Nets and Sub-nets, Convergence of a net, Ultrafilters, Partially Ordered Sets and Filters, Convergence of a filter, Ultrafilters, Basis and Subbase of a filter, Nets and Filters in Topology. Complete Metric Spaces and Function Spaces: Complete Metric Spaces, Baire Category Theorem, The Peano Space-Filling Curve, Hahn-Mazurkiewicz Theorem (statement only). Compactness in Metric Spaces, Equicontinuity. Pointwise and Compact Convergence, The Compact-Open Topology, Stone-Weierstrass Theorem, Ascoli's Theorem, Baire Spaces, A Nowhere Differentiable Function. An Introduction to Dimension Theory, Topological notion of (Lebesgue) dimension.

##### **4.8.1 Books for Reference in Advanced Topology - II**

1. Munkres, J. R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi.
2. Dugundji, J., Topology, Allyn and Bacon.
3. Simmons, G. F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
4. Kelley, J. L., General Topology, Van Nostrand Reinhold Co., New York.
5. Bourbaki, N., Topologie Generale.
6. Hocking, J. and Young, G., Topology, Addison-Wesley Reading.
6. Steen, L. and Seebach, J., Counter Examples in Topology, Holt, Reinhart and Winston, New York.

#### **4.9 S. P.- 4. Advanced Functional Analysis - II: Marks 20(I.A.)+55(S.E.) [Credit 6]**

Open Mapping Theorem. Closed Graph Theorem. Contraction Operator, Banach Fixed Point Theorem. Uniform Convexity, Strict Convexity. Approximation in normed spaces, best approximation- existence and uniqueness. Uniform approximation in  $C[a,b]$ . Haar Uniqueness Theorem. Spectral properties

of bounded linear operators. Banach Algebra and spectral properties of its elements. Compact linear operators on normed spaces and their spectral properties. Krein-Milman Theorem and its Applications, Fourier series, summability kernels, convergence tests. Fourier transforms, Fourier Inversion and Plancherel theorem.

#### **4.9.1 Books for Reference in Advanced Functional Analysis - II**

1. Rudin, W., Real and complex analysis, McGraw-Hill.
2. Rudin, W., Functional analysis, McGraw-Hill.
3. Conway, J. B., A course in functional analysis, GTM (96), Springer-Verlag.
4. Yosida, K., Functional analysis, Springer-Verlag.
5. Katznelson, Y., An introduction to harmonic analysis, Dover Publications.
6. Stein, E. M. and Shakrachi, R., Fourier Analysis: An Introduction, Princeton Lectures in Analysis.
7. Hernez, E. and Weiss, G., A first course on wavelets, Studies in Advanced Mathematics, CRC Press.
8. Kelley, J. L. and Namioka, I., Linear Topological Spaces, D.Van Nostrand Company.
9. Aliprantis, C. D., Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.
10. Goffman, C. and Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi.
11. Taylor, A. E., Introduction to Functional Analysis, John Wiley and Sons, New York.

### **4.10 S. P.- 5. Operations Research-II: Marks 20(I.A.)+55(S.E.) [Credit 6]**

#### **4.10.1 Sequencing**

Sequencing problems, Solution of sequencing problems, Processing  $n$  jobs through two machines, Processing  $n$  jobs through three machines, Optimal solutions, Processing of two jobs through  $m$  machines, Graphical method of solution, Processing  $n$  jobs through  $m$  machines.

#### **4.10.2 Project Scheduling and Network Analysis**

Project scheduling by PERT and CPM, Construction of a network, Fulker-son's rule, Errors and dummies in Network, Critical path analysis, Forward and backward pass methods, Floats of an activity, Project costs by CPM , Crashing of

an activity, Crash-cost slope, Time-cost trade off, Solution of network problems using Simplex technique. Time estimates for PERT, Probability of completion of a project within a scheduled time.

#### **4.10.3 Replacement Models**

Replacement problem, Types of replacement problems, Replacement of capital equipment that varies with time, Replacement policy for items where maintenance cost increases with time and money value is not considered, Money value, Present worth factor (pwf), Discount rate, Replacement policy for item whose maintenance cost increases with time and money value changes at a constant rate, Choice of best machine, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems.

#### **4.10.4 Inventory Problems**

Introduction, Inventory problems, Inventory parameters, Variables in inventory problems, Controlled and uncontrolled variables, Classification of inventory models, Deterministic elementary inventory models, Economic lot size formula and its properties, Problems.

#### **4.10.5 Books for Reference in Operations Research - II**

1. S. D. Sharma, Operations Research.
2. Kanti Swarup, P. K. Gupta and Manmohan, Operations Research.
3. Sasieni Maurice, Arther Yaspan, Lawrence Friedman, OR methods and Problems.
4. H. S. Taha, Operations Research.
5. T. L. Satty, Operations Resarch.

### **4.11 S. P.- 6. Fluid Mechanics-II: Marks 20(I.A.)+55(S.E.) [Credit 6]**

#### **4.11.1 Irrotational Motion in Two Dimensions**

General motion of a cylinder in two dimensions. Motion of a cylinder in a uniform stream, Liquid streaming past a fixed circular cylinder and two coaxial cylinders. Equations of motion of a circular cylinder. Circulation about a moving cylinder. Conjugate function. Elliptic cylinder. Liquid streaming past a fixed elliptic cylinder. Elliptic cylinder rotating in an infinite mass of liquid at rest at infinity. Circulation about an elliptic cylinder. Kinetic energy. Blasius theorem and its application. Kutta and Joukowski theorem, D'Alemberts paradox. Application of conformal mapping.

#### 4.11.2 Viscous Flow

Navier-Stokes equations, Vorticity and circulation in viscous fluids. Reynolds number, Boundary conditions. Flow of a viscous fluid with free surface on an inclined plane. Flow between parallel plates. Flow through pipes of circular, elliptic section under constant pressure gradient. Laminar flow between concentric rotating cylinder. Steady motion of a viscous fluid due to a slowly rotating sphere. Unsteady motion of a flat plate. Pulsatile flow between parallel surfaces. Prandtl's concept of boundary layer. Boundary layer flow along a flat plate. Momentum and energy integral equation for the boundary layer. Von Karman Pohlhausen method. Turbulence, Calculation of Turbulent BL.

#### 4.11.3 Books for Reference in Fluid Mechanics - II

1. Prandtl, L., Essential of fluid dynamics, Springer.
2. White, F. M., Viscous Fluid Flow, McGraw Hill.
3. Panton, R. L., Incompressible Flow, John Wiley and Sons.
4. Rosenhead, L., Laminar Boundary Layer, Dover.
5. Sherman, F. S., Viscous Flow (McGraw Hill).
6. Pai, S. I., Viscous Flow Theory, D. Van Nostrand.
7. Schlichting, H., Boundary Layer Theory, Springer.
8. Chorlton, F., Text Book of Fluid Dynamics, CBS Pub.
9. Love, A. E., A treatise on mathematical theory of elasticity, McGraw Hill Book Co.
10. Kondepudi, D. and Prigogine, I., Modern thermodynamics, John Wiley and Sons, Inc.
11. Landau, L. M. and Lifshitz, E. M., Fluid Mechanics, Butterworth Heine-mann.

#### 4.12 S. P.- 7. Nonlinear Differential Equations and Dynamical Systems - II: Marks 20(I.A.)+55(S.E.) [Credit 6]

##### 4.12.1 Dynamical System

Fixed points, periodic points, orbits, stable and unstable sets, Logistic and other noninvertible maps, circle map, centre sets, symbolic dynamics, topological conjugacy, structural stability, Chaos, period doubling cascades, pitchfork, saddle node, transcritical bifurcations, Hopf bifurcation, bifurcations in ODE, Poincare sequence, Homoclinic paths, Horseshoe map, toral automorphisms, chaos in non-linear ODE.



#### **4.12.2 Books for Reference in Nonlinear Differential Equations and Dynamical Systems - II**

1. D. W. Jordan and P. Smith, Nonlinear Ordinary Differential Equations, OUP.
2. E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, McGraw Hill.
3. R. L. Devaney, An Introduction to Chaotic Dynamical Systems, Westview Press, 2003.
4. B. Hasselblatt and A. Katok, A first Course in Dynamics, CUP.
5. R. A. Holmgren, A first course in discrete dynamical systems, Springer.
6. Alligood, Sauer and York, Chaos, an introduction to dynamical systems, Springer.
7. K. Falconer, Foundation to fractal geometry, CUP.

#### **4.13 S. P.- 8. Theory of Relativity, Astrophysics and Cosmology - II: Marks 20(I.A.)+55(S.E.) [Credit 6]**

##### **4.13.1 Cosmology**

Hubble's law, Cosmological principles. Weyl's postulate. Derivation of Robertson-Walker metric. Hubble and deceleration parameters. Redshift. Redshift versus distance relation. Angular size versus redshift relation and source counts in Robertson-Walker space-time, cosmological distances.

Friedmann models. Fundamental equations of dynamical cosmology. Critical density. Closed and open Universes. Age of the universe. Matter dominated era of the Universe. Einstein-de-Sitter model. Particle and event horizons. Distance-redshift relation. Early universe, Jeans mass, inflationary scenario, Eddington-Lamaitre models with  $\Lambda$ -term. Perfect cosmological principle. Steady state Cosmology.

Cosmology-Mach's principle, Einstein modified field equations with cosmological term. Static Cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe. References:

##### **4.13.2 Books for Reference in Theory of Relativity, Astrophysics and Cosmology - II**

1. W. M. Smart, Text book of Spherical Astronomy.
2. K. D. Abhyankar, Astrophysics: Stars and Galaxies, Tata McGraw Hill Publication.

3. Martin V. Zombeck, Handbook of Space Astronomy and Astrophysics, Cambridge University Press.
4. J. V. Narlikar, The Structure of the Universe, Prentice Hall.
5. T. Padmanabhan, An Invitation to Astrophysics, World Scientific.
6. C. E. Weatherburn, An Introduction to Riemannian Geometry and the tensor Calculus, Cambridge University Press.
7. H. Stephani, General Relativity: An Introduction to the theory of the gravitational field, Cambridge University Press.
8. A. S. Eddington, The Mathematical Theory of Relativity, Cambridge University Press.
9. J. V. Narlikar, General Relativity and Cosmology, The Macmillan Company of India Limited.
10. R. Adlev, M. Bazin, M. Schiffer, Introduction to general relativity, McGraw Hill Inc.
11. B. F. Shutz, A first course in general relativity, Cambridge University Press.
12. S. Weinberg, Gravitation and Cosmology: Principles and applications of the general theory of relativity, John Willey and Sons Inc.
13. R. K. Sachs and H. Wu., General Relativity for Mathematician, Springer Verlag.
14. L. D. Landau and E. M. Lifshitz, The classical theory of Fields, Pergamon Press.
15. S. K. Bose, Introduction to general relativity, Wiley Eastern, New Delhi.
16. N. Duric, Advanced Astrophysics, Cambridge University Press.
17. W. Kundt, Astrophysics: A new approach, Springer.
18. P. Lena, F. Lebrun and F. Mignard, Observational Astrophysics, Springer.
19. N. Straumann, General Relativity and Relativistic Astrophysics, Spinger-Verlog.
20. T. Padmanabhan, Theoretical Astrophysics (Vol. I, II and III), Cambridge University Press.

#### **4.14 Project/Field Studies: Marks 100 [Credit 8]**

(i)Project/Field Studies will be made by the students under the guidance of the teacher(s) of the Department, and on the basis of subject offered to the students of 4th Semistar.

(ii)Dissertation of the Project/Field Studies will be prepared by individual student and the same be submitted to the HOD after countersigned by the concerned teacher(s).